Section A

Answer ALL questions: **(10 x 2 = 20)**

1. Show that $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.

2. Prove that $\cos h^2 x - \sin h^2 x = 1$.

3. If α and β are the roots of the equation $2x^2 + 3x + 5 = 0$, find $\alpha+\beta$, $\alpha\beta$.

4. Find $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ if $u = x^2 + y^2$.

5. Integrate e^{5x+2} with respect to x.

6. Prove that $\displaystyle\int_a^b f(x)dx = -\int_b^a f(x)dx$.

7. Solve $p^2 - 3p + 2 = 0$.

8. Solve $px + qy + pq = z$.

9. Write the Newton-Raphson formula.

10. Write the Simpson's $\dfrac{1}{3}$ rule.

Section B

Answer any FIVE questions: **(5 x 8 = 40**

11. Express $sin^7\theta$ in a series of sines of multiplies of θ.

12. Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix}$.

13. Solve the equation $27x^3 + 42x^2 - 28x - 8 = 0$ whose roots are in geometric progression.

14. Verify Euler's theorem for the function $u = x^2 + y^2 + 2xy$.

15. Evaluate $\int x^4 e^{2x} dx$, using Bernoulli's formula.

16. Solve: $(D^2 + 5D + 6)y = e^{-2x} + \sin x$.

17. The velocity of a particle at distance S from a point on it's path is given by the following table

S(ft)	0	10	20	30	40	50	60
V(ft/s)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft using Trapezoidal rule..

18. Evaluate $\int_0^1 \dfrac{dx}{1+x^2}$ by taking 4 equal parts using (i) Simpson's $\dfrac{1}{3}$ rule (ii) Simpson's $\dfrac{3}{8}$ rule.

Section C
Answer any TWO questions: **(2 x 20 = 40)**

19. a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and also find A^{-1}.

    b) Separate into real and imaginary parts $\tan^{-1}(x + i\,y)$.   **(12+8)**

20. a) Solve equation $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

    b) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ is $4a\cos\dfrac{\theta}{2}$.   **(10+10)**

21. a) Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$

    b) Find the area of the surface of a cone whose semi vertical angle is α and base a circle of radius a.   **(10+10)**

22. a) Solve $\left(D^2 - 5D + 6\right)y = x^2 + 3$

    b) Solve $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} - 3y = \sin t$ given $y = \dfrac{dy}{dt} = 0$ when $t = 0$.   **(8 + 12)**